ALIASING AND ANTI-ALIAS FILTERS

PRECISION FILTERS, INC.

INTRODUCTION

The analysis of continuous analog signals by digital signal processing equipment requires that the analog signal be converted into a sampled, digitized record of finite length. Such a system is depicted in Figure 1 where an N-bit A/D converter is used to digitize an analog input f(t).

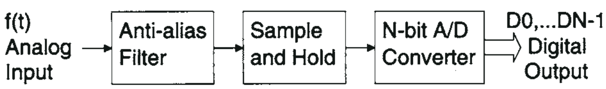

Figure 1 Block Diagram of a Data Conversion System

A point of great practical importance is that if the sampled data points are taken equally spaced in time (fixed conversion rate), then this same identical sampled data record could have come from other signals—aliases.

The insidious, unavoidable nature of this fact is illustrated in Figure 2 where the sampled data points are taken at a sampling rate of 5 Hz. The sampled data points of the 4 Hz sine wave are indistinguishable from sampled data points of the 1 Hz sine wave. The only way to prevent the 4 Hz sine wave from producing an alias at 1 Hz is to filter it out before sampling as shown in Figure 1.

ALIASES

Two frequencies, f_1 and f_2, are said to be aliases of each other if sampled data points of their corresponding sinusoids cannot be distinguished. This occurs if there exists an integer n such that $f_1 = nF_s \pm f_2$, where F_s is the sampling frequency. Another way of viewing this phenomena is that the sampled data points of every frequency in the frequency spectrum, no matter how high, are indistinguishable from the sampled data points of some frequency in the frequency interval from dc to $F_s/2$. The aliases of f are $nF_s \pm f$ where f lies in the interval between $-F_s/2$ and $F_s/2$, and n is an integer. The frequencies in this interval are referred to as principal aliases and the limit of this interval $F_s/2$ is referred to as the Nyquist or folding frequency.

SPECTRUM FOLDING

Aliasing is sometimes referred to as spectrum folding because the pattern of aliases corresponds to folding up the frequency axis. In Figure 3a, the frequency axis is marked off linearly in intervals which are multiples of the fold frequency $F_s/2$. Also labeled are a number of frequencies which are aliases of f_a. Aliases of $f_a = f_a, f_b, \ldots f_g = nF_s \pm f_a$. The frequency axis is then folded at multiples of the fold frequency as depicted in Figure 3b. Finally, in the completely folded spectrum, Figure 3c, the aliases are superimposed on each other and are indistinguishable.

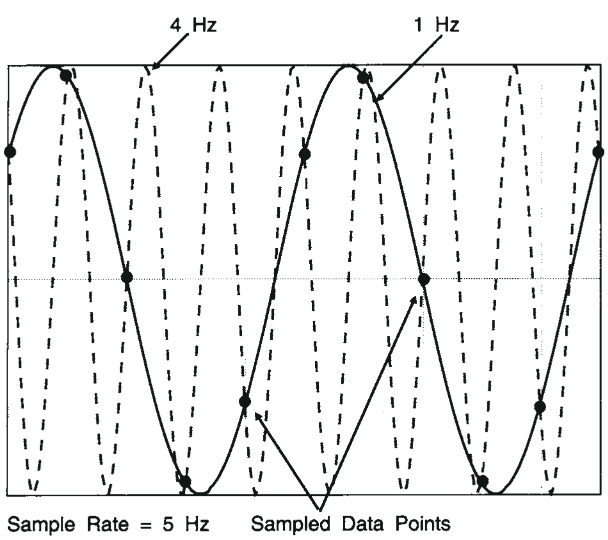

Figure 2 Signals with Identical Sampled Data Points

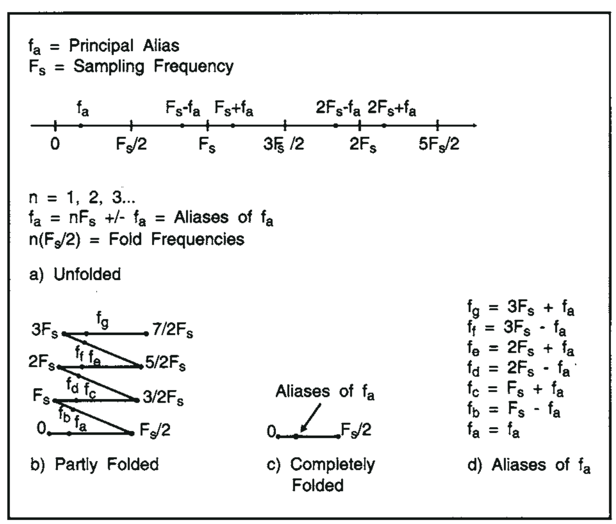

Figure 3 Spectrum Folding or Aliasing

SAMPLING THEOREM

The Sampling Theorem states that if a function of time, f(t), contains no frequency higher than B hertz, it is completely determined by a sampled data record with a sampling frequency of 2B. Restating the Sampling Theorem in a manner which emphasizes the problem of aliasing:

1. The sampling frequency, F_s, must be at least twice the highest frequency contained in the signal to prevent the signal from being folded into itself.

2. No interference higher than $F_s/2$ can exist with the signal, because it would be folded into the signal.

It is the second part which in practice causes the problem, for aliases above $F_s/2$ often do exist with the signal and require a filter to attenuate them to an acceptable level. Practical filters, however, do not have infinite attenuation slopes, and so a higher sampling frequency is required.

ANTI-ALIAS FILTERS

An anti-alias filter is used in front of an A/D converter to attenuate the aliases without attenuating the signal. (See Figure 1.)

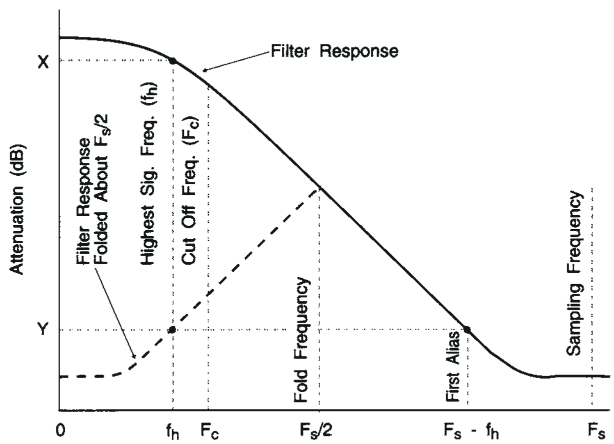

Figure 4 Anti-alias Filter Folded about Fs/2 (Linear Frequency Axis)

The procedure to properly set up the anti-aliasing filter for data acquisition is as follows (Refer to Figure 4):

- The filter cutoff frequency is set so that the highest frequency in the signal, f_h, is attenuated no more than X dB.
- The sampling frequency is set high enough to permit the filter to attenuate the first alias of f_h, (F_s - f_h), by at least Y dB.

Figure 5 compares the plots of the attenuation of the first alias of f_h versus the sampling frequency for three filter types: a 6 pole, 6 zero elliptic low-pass filter; an 8 pole, 6 zero elliptic low-pass filter; and an 8 pole Butterworth low-pass filter. If all aliases were to be attenuated by at least 80 dB and the highest frequency contained in the signal f_h was to be attenuated by no more than 0.1 dB, the sampling frequency would be:

- 5.00 f_h for an 8 pole Butterworth,
- 3.13 f_h for the Precision Filters 6 pole, 6 zero LP1, and
- 2.56 f_h for the Precision Filters 8 pole, 6 zero LP8 filter.

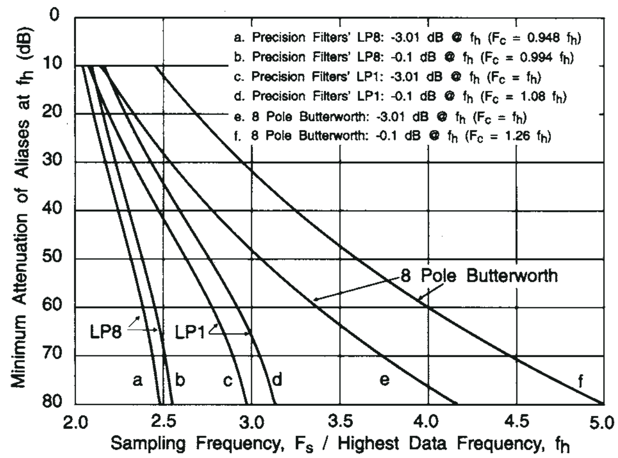

Figure 5 Minimum Attenuation of Aliases vs. Sampling Frequency

The advantages offered by the 6 pole, 6 zero and the 8 pole, 6 zero elliptic low-pass filters over the 8 pole Butterworth low-pass as anti-alias filters are:

- Less attenuation of the signal
- More attenuation of aliases
- Lower sampling frequency
- More data bandwidth
- Less data to be stored

FILTER OUTPUT NOISE

All signals at the filter output including the broadband filter output noise will be folded into the data band by the A/D converter; so the filter's broadband output noise can limit the dynamic range of the system. It is of little use to specify the filter output noise with a 100 kHz or 1 MHz detector bandwidth when the A/D converter can alias the spectrum above 10 MHz into the data band. 20 MHz is a practical detector bandwidth for measuring the output noise of active filters, for above 20 MHz the active components in the filter will limit the noise.

A Precision Filters' filter output noise is less than 250 μV in the 20 MHz bandwidth.

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