Anti-Aliasing Filters Reduce Errors in Data Acquisition

Aliasing is the corruption of in-band signal by out-of-band signals during A/D conversion. Low-pass anti-aliasing filtering before A/D conversion is the only means to prevent aliasing in data acquisition for test measurements.

Introduction

When analog signals are sampled by A/D converters, the resulting digital data record can be corrupted by signals outside of the bandwidth of interest and cannot be distinguished from the in-band signals. Termed aliasing, the potential corruption is undetectable.

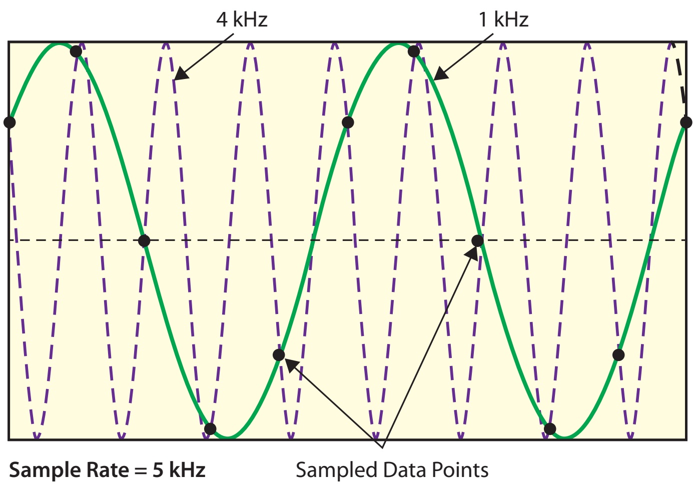

Figure 1: 4 kHz Alias of 1 kHz Signal at 5 kHz Sample Rate

As illustrated in Figure 1, where sampled data points are taken at a sampling frequency, F_S, of 5 kHz. The sampled data points of the 4 kHz sine wave are indistinguishable from the sampled data points of the 1 kHz sine wave. The only way to prevent the 4 kHz sine wave from producing an alias at 1 kHz is to use an analog anti-alias filter to attenuate the 4 kHz signal at the A/D converter input as shown in Figure 2.

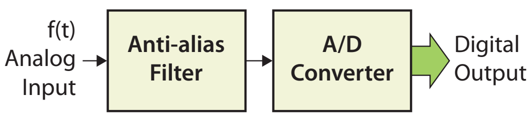

Figure 2: Block Diagram of Data Conversion System

Aliases

Two frequencies (f_1 and f_2) are said to be aliases of each other if sampled data points of their corresponding sinusoids cannot be distinguished. This occurs if there exists a positive integer, n, such as $f_1 = nF_S \pm f_2$. Sampled data points of every frequency in the spectrum, no matter how high, will have the equivalent sampled data points of some frequency in the interval from DC to $F_S/2$.

The aliases of a given frequency in the signal of interest, f_a, lying in the interval from DC to $F_S/2$ are $nF_S \pm f_a$. The frequencies in this interval are referred to as principal aliases and the limit of this interval $F_S/2$ is termed the Nyquist or folding frequency.

Aliasing is also referred to as spectrum folding because the pattern of aliases corresponds to the folding up of the frequency axis. In Figure 3, the frequency axis is marked off linearly in intervals that are multiples of the folding frequency. Also labeled on this axis are a number of frequencies that are aliases of one another, f_a, f_b, f_c, … f_g, $= nF_S \pm f_a$. If the frequency axis is folded at multiples of $F_S/2$, then the set of aliases f_a, f_b, f_c, … f_g are superimposed on each other and are indistinguishable.

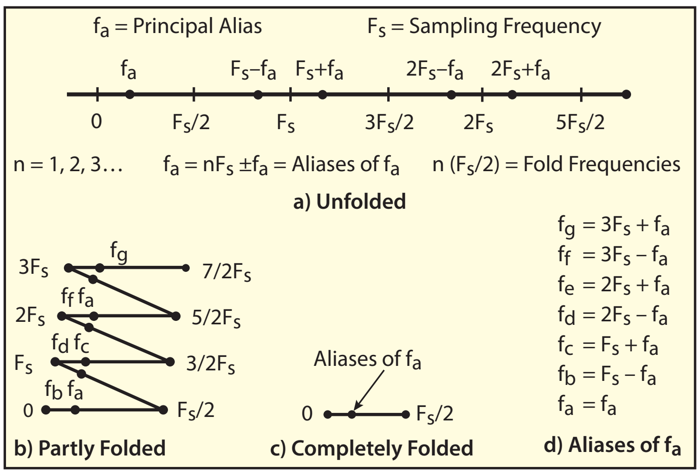

Figure 3: Aliasing as Spectrum Folding at Multiples of $F_S/2$

Attenuation of Aliases

Low-pass filtering, in addition to removing out-of-band energy that could corrupt in-band data, serves to band-limit the signal prior to subsequent sampling. The sampling frequency of the digitizer must then be set high enough to assure adequate attenuation of signals that could alias into the pass-band of the filter. Another consideration on setting the sampling frequency is the allowable error in resolving the amplitude of the waveform up to the highest frequency of interest. Here, however, we restrict our discussion to determining the minimum sampling frequency required to achieve the desired attenuation of aliases.

From our prior discussion, it is clear that any frequency that exists higher than $F_S/2$ will fold into the band between DC and $F_S/2$. Given this, a low-pass filter is needed to attenuate these higher frequencies to an acceptable level. In practice, the theoretical sampling frequency must be increased to account for the fact that actual filters do not have infinite attenuation slopes. The low-pass filter in front of the A/D converter must be set to attenuate signals that will alias without significantly attenuating the signal itself.

The procedure for setting the cutoff frequency of the filter is as follows:

1. The filter cutoff is set so that the highest frequency in the signal, fx, is attenuated by not more than X dB.
2. Referring to Figure 4, the sampling frequency is set high enough to permit the filter to attenuate the first alias of fx, fy = FS - fx, by at least Y dB. The sampling frequency FS may be expressed in terms of fx and its first alias as:

$$F_s = \left(\frac{f_y}{f_x} + 1 \right) f_x$$

As an example, we assume the filter cutoff frequency is $f_x = F_{3dB}$ and that 80 dB minimum attenuation of its first alias, f_y, is required. We define F_{80dB} as the frequency of the filter where 80 dB of attenuation is reached. With these assumptions, the equation for setting the sampling frequency becomes:

$$F_s = \left(\frac{F_{80dB}}{F_{3dB}} + 1 \right) F_{3dB}$$

Values of F_{80dB} for various filters are provided in Table 1. The resulting sampling frequency when using an 8-pole Butterworth (BU8) filter would be $F_s = 4.16{*}F_{3dB}$. Similarly, for the sharper Precision Filters LP8F under the same constraints, $F_S = 2.75{*}F_{3dB}$, which is a 34% reduction in sampling rate compared to the BU8. The Precision Filters LP6F 6-pole filter results in $F_S = 3.61{*}F_{3dB}$, a 13% reduction in sampling rate when compared to the BU8.

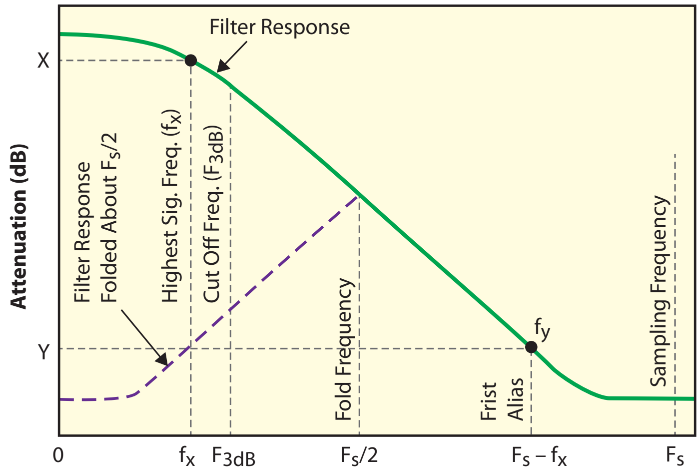

Figure 4: Anti-Alias Filter with Spectrum Folding about $F_s/2$ (Linear Frequency Axis)

Conclusion

Aliasing is an inescapable attribute of A/D conversion and digital data acquisition. Without proper intervention, digital data can easily become meaningless without any indication. Barring advance knowledge of the full spectrum of the analog signal, the only strategy to prevent in-band signal corruption by aliasing is by using low-pass anti-aliasing filtering prior to the A/D converter. The low-pass filtering must be carefully set both to minimize attenuation of pass-band signals of interest and maximize attenuation of out-of-band aliasing frequencies. Further, the sharpness of the applied filter — as with those offered by Precision Filters — has a direct effect on both the quality of in-band signal fidelity as well as the required sampling frequency.

Low-Pass Filter		**Pass-Band Amplitude Response (f/F_{3dB})**			**Transition Region Amplitude Response (f/F_{3dB})**			
Filter	**Description**	**-1%**	**-5%**	**F_{3dB}**	**-20 dB**	**-40 dB**	**-60 dB**	**-80 dB**
LP8F	PFI 8-Pole Maximally Flat Elliptic	0.86	0.91	1	1.21	1.44	1.64	1.75
LP6F	PFI 6-Pole Maximally Flat Elliptic	0.78	0.87	1	1.38	1.86	2.32	2.61
BU8	8-Pole Butterworth	0.78	0.87	1	1.33	1.78	2.37	3.16

Table 1: Comparison of Low-Pass Filter Properties

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