Signal Conditioner Self-Noise: Characterization and Specification

Thomas P. Gerber, Douglas R. Firth, Alan R. Szary

Overview

- While the excitation, amplification, and filtering provided by analog signal conditioners can improve the overall signal-to-noise ratio (SNR) of low-level sensor measurements, the components that make up the conditioner introduce additional noise to the measurement circuit.
- Though generally small compared to external and sensor noise, the signal conditioner's "self-noise" should be properly characterized and specified.
- Proper characterization of self-noise requires an understanding of how noise is modeled and analyzed in an electrical circuit. Key concepts include noise level quantification, combination of independent noise sources, spectral noise density, and noise-equivalent bandwidth.
- Precision Filters employs a simplified model of noise generation within a signal conditioner to provide product specifications that can be used for quick estimates of self-noise SNR.

PRECISION FILTERS, INC.

Transducer Conditioning Systems ■ Filter/Amplifier Systems ■ Signal Switching Systems

1. Introduction

A signal conditioner is often used to measure low-level sensor signals, providing necessary excitation, amplification, and filtering. When properly implemented, analog signal conditioning can significantly suppress noise from the sensor and external environment. However, the conditioner's components add their own noise to the measurement circuit. Though in most applications external and sensor noise will dominate this "self-noise" (also referred to as intrinsic noise), the measurements engineer should understand the level of noise that is generated by their signal conditioner. Characterizing the self-noise from a circuit with multiple gain stages and an active filter is not straightforward, and there is no established standard for its specification. This paper summarizes how self-noise is specified for Precision Filters' (PFI) signal conditioning products. An introduction to electrical noise analysis is presented first to establish a foundation for understanding how noise is modeled and measured. Noise specifications for PFI's signal conditioners are then explained, followed by some simple examples of how such specifications can be used to estimate the SNR of a sensor measurement.

2. Noise Characterization

The following sections introduce some key concepts related to the modeling and analysis of noise in an electrical circuit. The summary presented here is not meant to be exhaustive; rather, a basic framework is established to inform the subsequent discussion of PFI's noise specifications. Relevant mathematical details are covered in an appendix.

2.1. Noise Level: RMS and Peak-to-Peak Voltages

Internal noise in a signal conditioner is generated by the resistors and op-amps in the circuit. Though the generative mechanisms vary, all electrical noise is stochastic: the noise waveform is unpredictable and can only be described statistically. Noise sources are therefore primarily modeled as voltages that vary randomly with time.

An example of a white noise voltage waveform is illustrated in Figure 1. The instantaneous amplitudes are described by a normal probability distribution with zero mean and a standard deviation of 1 µV. A definition of the noise level is defined in terms of the properties of the distribution. Notice in Figure 1 that, with a zero-mean noise voltage (i.e. no DC offset), the standard deviation of the distribution equates to the root-mean-square (RMS) voltage, 1 μV_{rms} (see Appendix A.1).

A measure for the peak noise level is more ambiguous, as it must be defined in terms of an exceedance probability. A peak voltage can be estimated by multiplying the RMS voltage by a constant referred to as a crest factor. The industry-standard crest factor for noise analysis is 3.3. For zero-mean, normally distributed noise voltages, this means the probability of the noise falling outside the peak-to-peak range is 0.1%. With an RMS voltage of 1 μV_{rms}, the peak voltage is therefore ±3.3 μV_p and the peak-to-peak voltage is 6.6 μV_{pp}.

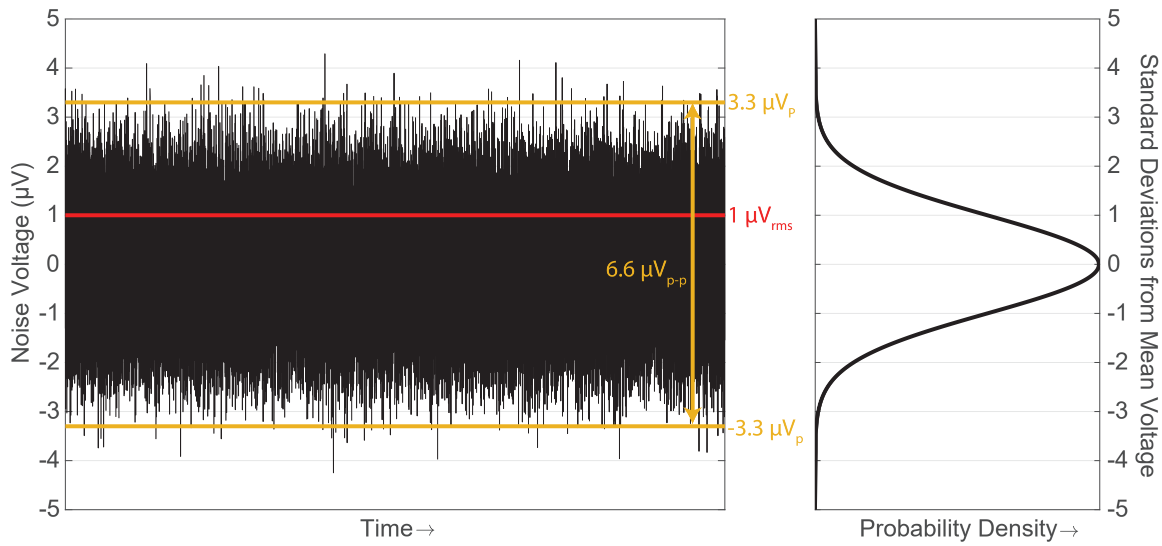

Figure 1. Example of a white-noise waveform (*left*) with a normal probability distribution of instantaneous voltages (*right*).

Noise specifications for electrical devices will often include measurements reported in units of μV_{rms} or μV_{pp}. It's worth noting that not all sources of circuit noise can be described by a normal probability distribution of voltages. Nevertheless, the definitions given above are still used to convert noise measurements from μV_{rms} to μV_p and μV_{pp}.

2.2. Combining Noise Sources

Circuit noise in a signal conditioner includes contributions from multiple components. Each noise source can generally be treated as independent and uncorrelated from all other sources. Consequently, the total noise voltage can be obtained by combining individual noise voltages in root-sum-square (RSS) fashion (see Appendix A.2). Formally, if E_i is the RMS voltage of the i^{th} noise source, then the total RMS noise voltage is given by

$$E_n = \sqrt{\sum_i E_i^2} \tag{1}$$

As a simple example, consider two noise sources, denoted $v_1(t)$ and $v_2(t)$, that produce the voltage waveforms shown in Figure 2A. Over the measurement interval, the corresponding RMS voltages E_1 and E_2 differ by a factor of 3 ($E_1 = 3E_2$). Figure 2B shows the combined noise waveform, $v_1(t) + v_2(t)$. The RMS voltage of the combination differs from E_1 by only 5%. In other words, the noise level of the combination is almost indistinguishable from that of the larger component E_1. For the general case of two sources, if E_1 is a factor α larger than E_2, their RSS combination is given by

$$E_n = \sqrt{1 + \frac{1}{\alpha^2}} E_1 \tag{2}$$

which shows that E_n will rapidly converge to E_1 as α gets large. This property of RSS combinations has important implications for noise analysis. In a system with multiple noise sources, if the RMS voltage of one source is known to be significantly larger (i.e. factor of ~3 or greater) than the others, it will dominate the noise signal and the others can often be neglected.

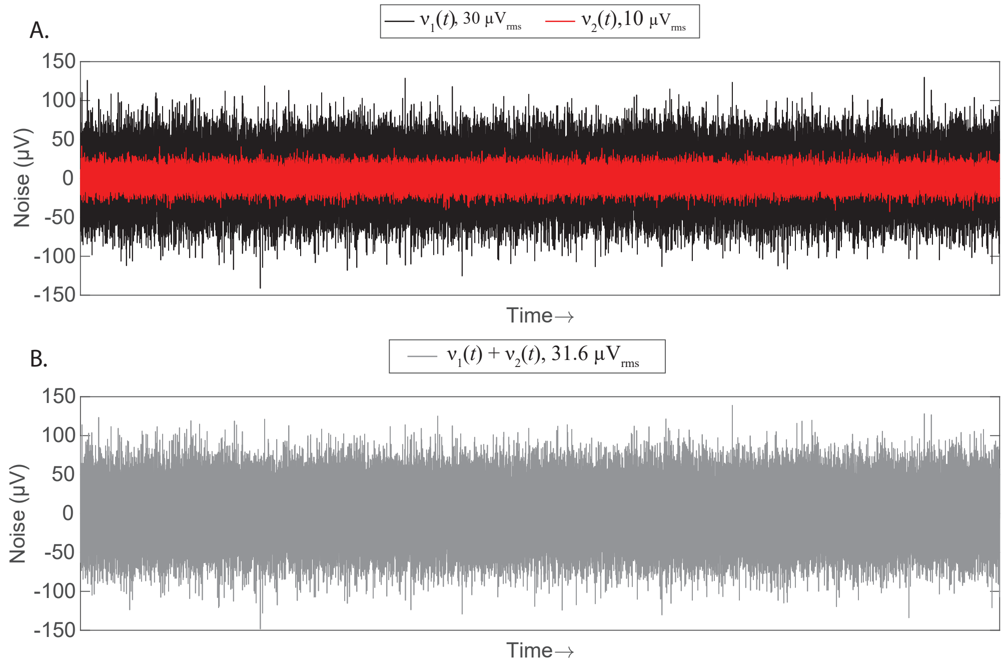

Figure 2. (A) Two independent noise sources, $v_1(t)$ and $v_2(t)$, with corresponding RMS voltages $E_1 = 30\ \mu V_{rms}$ and $E_2 = 10\ \mu V_{rms}$. (B) Combination of $v_1(t)$ and $v_2(t)$, with resulting RMS voltage $E_n = 31.6\ \mu V_{rms}$.

2.3. Noise Spectral Analysis

The preceding discussion addressed noise characterization in the time domain. However, noise specifications are commonly given in the frequency domain. This is because electrical noise is broadband: the most common noise sources have power across a range of frequencies, so the SNR of a measurement will depend on the bandwidth. For this reason, noise is commonly specified using spectral densities.

The noise spectral density, e_n, can be obtained via spectral analysis (e.g. FFT) of a measured noise voltage. It is computed as an RMS amplitude spectrum that is normalized by the frequency resolution of the measurement, and can be interpreted as the RMS voltage that would be measured if the noise were passed through a perfect (or "brickwall") 1 Hz bandpass filter centered at frequency *f*. The noise spectral density e_n is expressed in the following units[1]:

$$e_n(f) \equiv \left[\frac{\mathrm{V_{rms}}}{\sqrt{\mathrm{Hz}}}\right] \tag{3}$$

In terms of spectral density, two types of circuit noise are generated in a signal conditioner (Figure 3). A frequency-dependent "pink" noise (also referred to as "1/*f*" noise) occurs with a spectral density that can be modeled by

1 Noise spectral density can also be given in terms of power, which equates to e_n^2 with units of $\mathrm{V_{rms}^2/Hz}$.

$$e_{nf} = \frac{K}{\sqrt{f}} \tag{4}$$

where K is a reference RMS voltage that is determined by the design of the signal conditioner and the components that are used. Notice that K equates to the pink noise density at 1 Hz. Pink noise is inversely proportional to frequency (hence its description as "$1/f$" noise[2]): its importance diminishes with increasing frequency. When viewed on an oscilloscope, a voltage waveform with a noise spectral density defined by (4) would resemble the trace shown in Figure 4A.

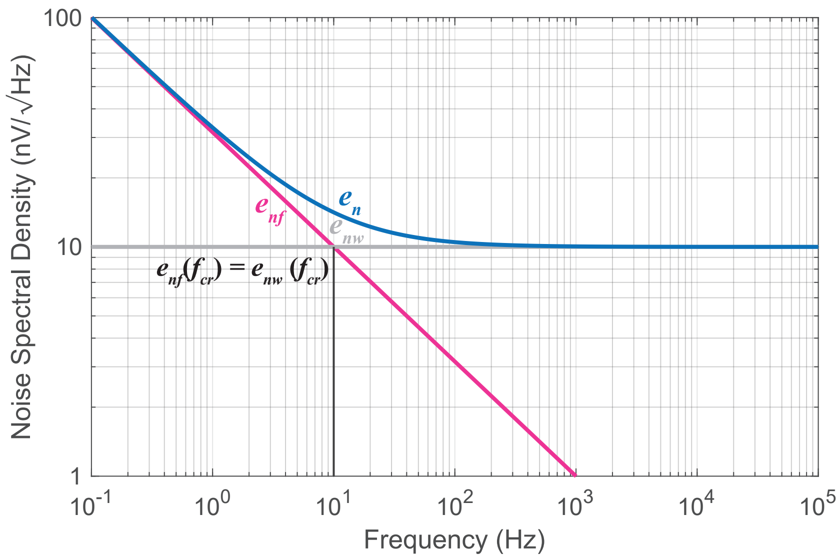

Figure 3. Noise spectral density plot showing pink noise (e_{nf}), white noise (e_{nw}), and their combination (e_n). The corner frequency f_{cr} (where $e_{nf} = e_{nw}$) is 10 Hz.

Signal conditioner circuits also generate white noise which, by definition, is independent of frequency (Figure 3):

$$e_{nw} = e_c \tag{5}$$

In (5), e_c is a constant white noise spectral density (e.g. $10\ \text{nV}/\sqrt{\text{Hz}}$ in Figure 3). A voltage waveform with a noise spectral density defined by (5) would resemble the trace shown in Figure 4B.

For a given circuit, the relative importance of pink and white noise is characterized by the corner frequency f_{cr}, defined as the frequency at which $e_{nf} = e_{nw}$ (10 Hz in Figure 3). Between (4) and (5), the corner frequency can be expressed as

$$f_{cr} = \left(\frac{K}{e_c}\right)^2 \tag{6}$$

Corner frequencies are typically between 1 Hz -100 Hz, depending on the type of op-amps used in the circuit.

2 Pink noise is *sensu stricto* $1/\sqrt{f}$ noise when defined as an amplitude spectral density; the term "$1/f$" noise refers to its corresponding power spectral density.

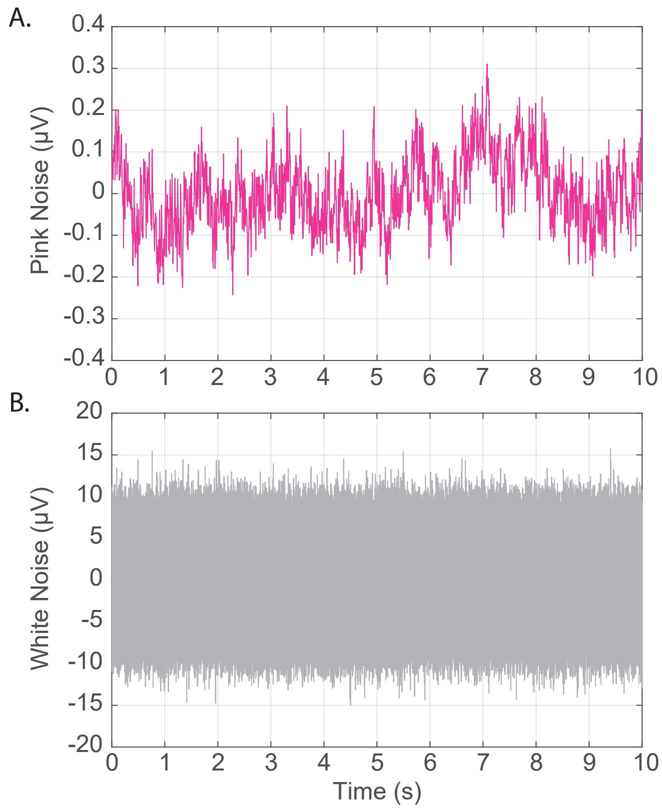

Figure 4. (A) Example time record (10 s) of pink noise over a bandwidth of 0.1 Hz to 100 Hz. (B) Example time record (10 s) of white noise over a bandwidth of 0.1 Hz to 100 kHz.

With (6), the RSS combination of pink and white noise yields a total noise density given by (see Appendix A.3)

$$e_n = e_c\sqrt{\frac{f_{cr}}{f}+1} \tag{7}$$

which is plotted in Figure 3. Note that at the corner frequency:

$$e_n = e_c\sqrt{2} \tag{8}$$

In a circuit whose noise is well-described by (7), the RMS noise over any bandwidth B defined by the frequency limits f_L and f_H is found via integration:

$$E_n = \sqrt{\int_{f_L}^{f_H} e_n^2 df} \tag{9}$$

Applying (9) to (7) gives

$$E_n = e_c\sqrt{f_{cr}\ln\left(\frac{f_H}{f_L}\right)+\left(f_H - f_L\right)} \tag{10}$$

Figure 5 illustrates the relationship defined in (10) by plotting[3] the RMS noise E_n vs. bandwidth (where $B = f_H - f_L$) for a fixed lower frequency bound of f_L = 1 Hz. Curves for three different corner frequencies are shown, along with the white-noise-only approximation (i.e. $f_{cr} \approx 0$).

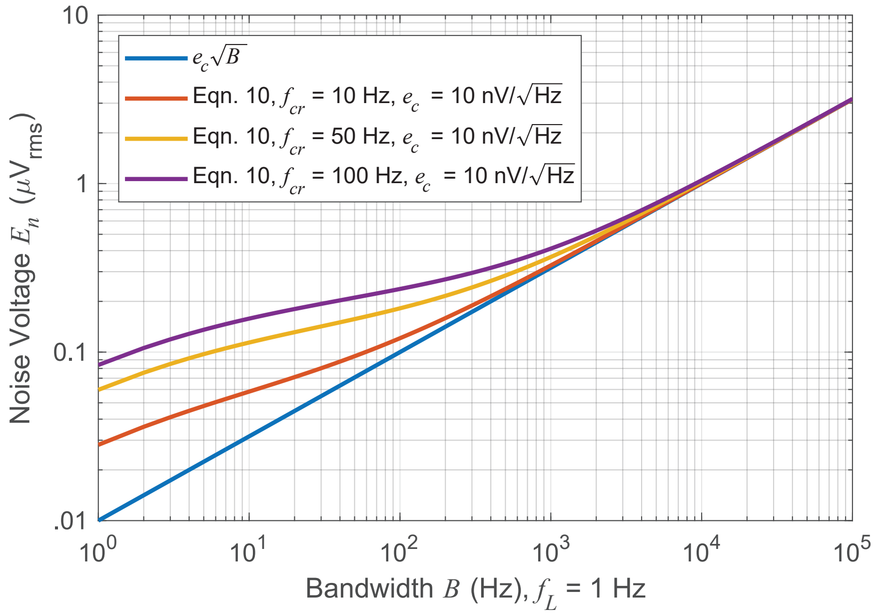

Figure 5. RMS noise voltage as a function of bandwidth with a lower frequency bound f_L of 1 Hz. Plots are shown for three corner frequencies (f_{cr} = 10 Hz, 50 Hz, 100 Hz).

The contribution of pink noise to the total RMS noise increases with f_{cr} but is only significant at low frequencies. For bandwidths in excess of 1 kHz, the total RMS noise is dominated by the white noise component such that

$$E_n \approx e_c\sqrt{B} \tag{11}$$

For this reason, signal conditioner circuit noise is commonly specified as a noise spectral density (e_n) at a reference frequency in the white-noise dominated part of the spectrum (i.e. >> f_{cr}, typically ≥ 1 kHz) so that (11) can be used to obtain a quick estimate of the total RMS noise for broadband measurements.

Alternatively, RMS (or peak-to-peak) noise E_n can be specified over a reference bandwidth. In this case, it's not uncommon for two frequency ranges to be reported: a low-frequency band where pink noise is important (e.g. .1 Hz – 10 Hz, where Eqn. 10 applies), and a broadband range (e.g. 10 Hz to 100 kHz, where Eqn. 11 holds) dominated by white noise.

3 Note that the y-intercept is defined by the noise voltage over a 1 Hz bandwidth with a lower bound of 1 Hz, which from (10) equates to: $E_n = e_c\sqrt{1+.693 f_{cr}}$

2.4. Noise Bandwidth

The noise spectral density e_n modeled by (7) and plotted in Figure 3 has no defined upper frequency bound. However, all signal conditioner circuits have an amplitude response that rolls off at a finite frequency, causing a corresponding roll-off of the white noise generated by the circuit. An example of a white-noise spectral density with a first-order amplitude roll-off is shown in Figure 6, where the cutoff frequency f_c – defined as the frequency at which the white noise spectral density e_{nw} is down by 3 dB – is 500 kHz.

To simplify broadband noise calculations and enable the use of (11), it is convenient to define an equivalent noise bandwidth B_n in terms of the cutoff frequency f_c. The noise bandwidth B_n is the bandwidth of an ideal brickwall response e_{nbw} that, when substituted into (11), yields the same RMS noise as the noise spectral density e_n integrated over the response of the amplifier (Figure 6). That is,

$$E_n = e_c\sqrt{B_n} = e_c\sqrt{kf_c} \tag{12}$$

where k is the brickwall correction factor. For the first-order response shown in Figure 6, the correction factor is 1.57 (analysis and derivation given in Appendix A.4). In general, $1 \le k \le 1.57$, with k approaching 1 for higher order (i.e. sharper) responses. If the cutoff frequency f_c, white noise density e_c, and brickwall correction factor k of the measurement circuit are known, then (12) can be used to estimate the broadband RMS noise.

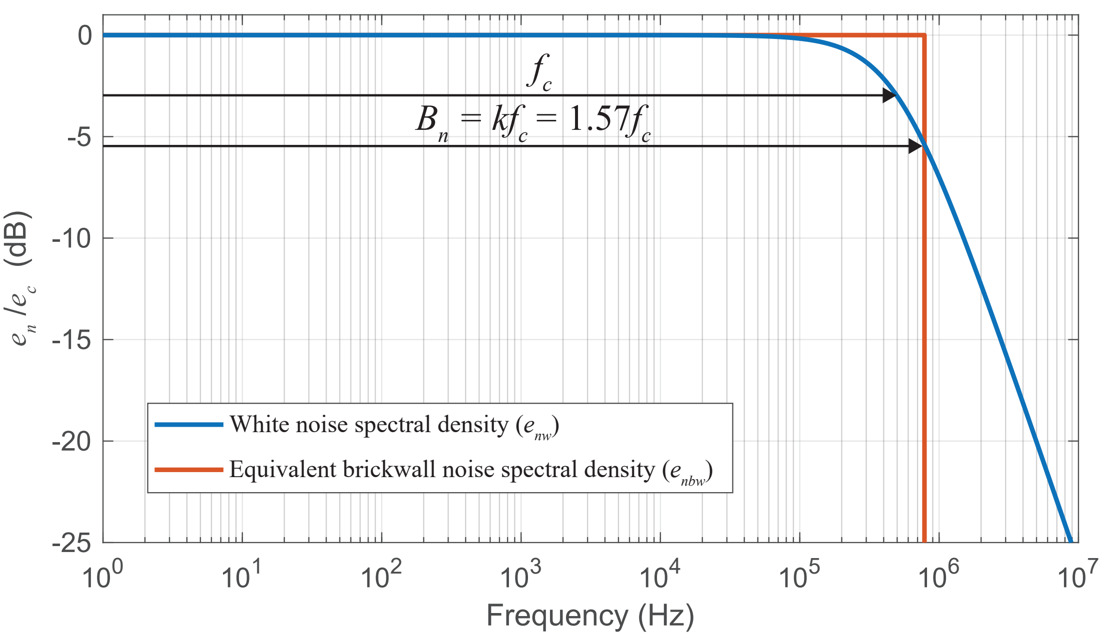

Figure 6. Roll-off of the white noise spectral density at high frequency. The first-order response for e_{nw} (see Appendix A.4) is plotted for a -3 dB cutoff frequency f_c of 500 kHz. The equivalent brickwall response, e_{bnw}, is shown for comparison, where the first-order brickwall correction factor k is 1.57.

In the preceding discussion, it was assumed that the frequency roll-off of white noise generated by the signal conditioner was caused only by the band-limiting effect of the amplifier in the circuit. If the white noise is passed through a low-pass filter with a lower cutoff frequency than the amplifier, then the bandwidth of the filter will determine the total RMS noise. In this case (12) still applies, but with f_c and k defined by the properties of the filter. Brickwall correction factors for commonly used 4th and 8th order filters in PFI's signal conditioner products are given in Table 1.

Table 1. Brickwall Correction Factors for PFI Filters

Filter Type	Filter Order	Brickwall Correction Factor (*k*)
Butterworth†	4	1.026
Butterworth†	8	1.007
Bessel	4	1.047
Bessel	8	1.044
Pulse*	4	1.106
Pulse*	8	1.076
Flat*	4	1.035
Flat*	8	1.013

**Proprietary filter design   †See Appendix A.4*

3. PFI's Noise Specifications

The concepts covered in the preceding section provide a sufficient basis for noise characterization in an electrical circuit. However, the analysis of noise in a signal conditioner is complicated by the fact that the circuit includes analog filters and multiple gain stages with several noise-generating components. The noise level measured on the signal conditioner output therefore represents contributions from multiple sources with different gains that cannot be easily isolated.

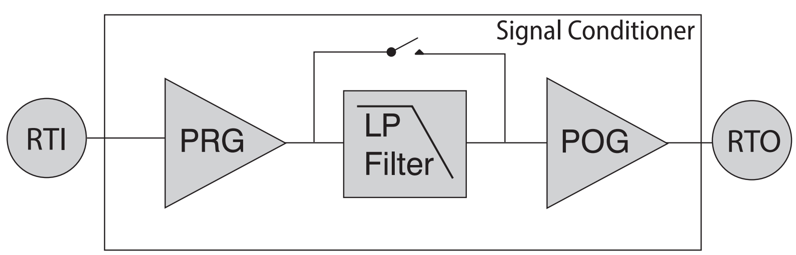

Figure 7. Simplified noise model used as the basis for PFI's signal conditioner noise specifications.

To obtain product specifications based on output measurements alone, PFI employs a simplified noise model that yields figures intended for quick signal-to-noise (SNR) estimates. The total self-noise in a PFI signal conditioner is partitioned into two components: an RTI (*referred-to-input*) noise source at the signal conditioner input, and an RTO (*referred-to-output*) noise source at the signal conditioner output. The RTI noise is amplified by the total gain of the system, which in most PFI filter-amplifiers is distributed between pre-filter (PRG) and post-filter (POG) gain stages.

The RTI noise is also passed through the low-pass filter (unless the filter is bypassed). The RTO noise can be interpreted as the noise that would appear on the signal conditioner output with unity gain and no filter in the signal path.

The following sections summarize how PFI uses this model to determine a standard noise specification for signal conditioner products, and explains how those specifications can be used to estimate noise levels in measurement applications.

3.1. RTI Noise

As a representative example, consider PFI's 28144 quad-channel wideband transducer conditioner. In the Input Characteristics listed on the 28144 Specification Sheet[4], the following noise specification is found:

Noise:

9 nV/$\sqrt{\text{Hz}}$ at 1 kHz and pre-filter gain > 64, typical

Notice that this is a spectral noise density at a single reference frequency of 1 kHz. The specification is further defined as applying to a conditioner setting in which the pre-filter gain (PRG) is greater than x64.

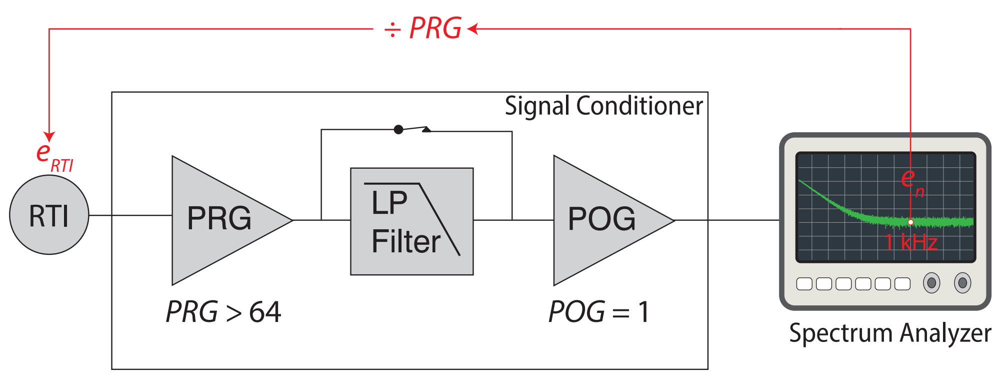

Figure 8. Illustration of the procedure used to specify the RTI noise spectral density that is listed as a standard input characteristic on PFI's specification sheets.

Figure 8 illustrates how this measurement is made and reported. With the signal conditioner input shorted to ground, the output channel is connected to a spectrum analyzer. The programmable PRG on the channel is set to a high value (> x64) while the POG is set to unity. The low-pass filter is bypassed, removing it from the signal path. The spectral density of the noise (e_n) on the channel at a reference frequency of 1 kHz – well within the white-noise dominated part of the spectrum for the 28144 – is then read out. This value is divided by the PRG to yield an RTI noise spectral density (e_{RTI}).

The discussion in Section 2.2 provides further insight into what this measurement represents. The

4 Available at https://pfinc.com/product-spec-sheets/

fact that independent noise sources combine in RSS fashion means that with PRG >> 1 and POG = 1, the measured noise is dominated by noise at the conditioner's input stage (i.e. upstream of the pre-filter gain stage). In other words, with a high PRG and POG = 1, the measured noise is input-dominated: the noise seen at the output is sourced almost entirely from the signal conditioner's input stage.

Because the RTI noise represents input-stage noise, it must be specified as a spectral density so that the effect of the low-pass filter on the total RTI noise level (E_{RTI}) can be accounted for in noise calculations, as explained in Sections 2.3-2.4.

3.2. RTO Noise

Again referring to PFI's 28144 Specification Sheet, the following information is listed in the Output Characteristics section:

Noise:
2.8 μV_{rms} RTI + 60 μV_{rms} RTO
3 Hz to 100 kHz

Notice that the output noise is expressed as a sum of RTI and RTO noise, and is a broadband specification (over the bandwidth 3 Hz to 100 kHz). Consider here the second term in the sum, the RTO noise. Figure 9 illustrates how this measurement is made and reported. With the signal conditioner input shorted to ground, the output channel is connected to a digital voltmeter (DVM). The programmable PRG and POG on the channel are both set to unity and the filter bypassed. The output is then read out over a band from 3 Hz (the lower limit of the DVM) to 100 kHz (the upper limit set by the measurement system bandwidth) and reported as an RMS voltage (E_n). This value is specified directly as the RTO noise, which for the 28144 equates to 60 μV_{rms}.

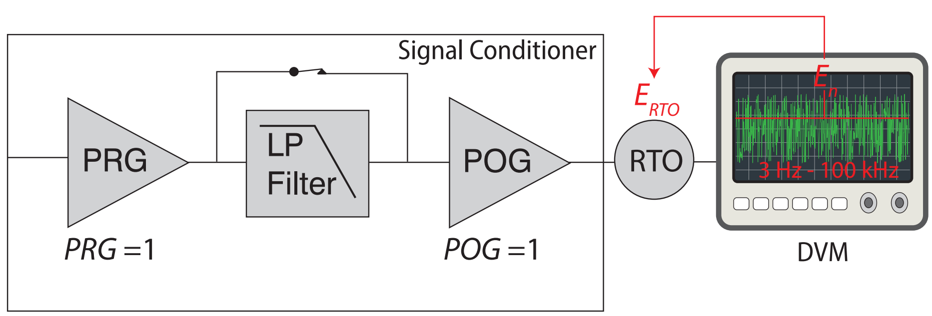

Figure 9. Illustration of the procedure used to specify the broadband RTO noise that is listed as a standard output characteristic on PFI's specification sheets.

3.3. Total Output Noise

The summation of the RTO noise with an RTI noise in the signal conditioner's output characteristics requires a few points of clarification. Firstly, the combination of the RTI and RTO sources is – as explained in Section 2.2 – an RSS sum, not an arithmetic sum. Also recognize that the 2.8 μV_{rms} equates to the RTI noise spectral density (defined in Section 3.1) converted to a broadband RMS noise over the specified ~100 kHz bandwidth[5]. And notice that the RTO measurement depicted in Figure 9 *includes* this input-stage RTI noise.

With the preceding in mind, the total output noise for the setup shown in Figure 9 can be estimated as

$$E_n = \sqrt{\left(2.8\ \mu\text{V}_{\text{rms}}\right)^2 + \left(60\ \mu\text{V}_{\text{rms}}\right)^2} \approx 60.1\ \mu\text{V} \approx \sqrt{\left(60\ \mu\text{V}_{\text{rms}}\right)^2}$$

The fact that the 60 μV_{rms} RTO includes the 2.8 μV_{rms} RTI is insignificant: the RSS combination approximates the actual (measured) RTO noise level. Contrast that with an input-dominated setup like that shown in Figure 8. Assuming a PRG of x128 with POG set to unity, the total output noise over the 3 Hz-100 kHz bandwidth can be estimated as

$$E_n = \sqrt{\left(128 * 2.8\ \mu\text{V}_{\text{rms}}\right)^2 + \left(60\ \mu\text{V}_{\text{rms}}\right)^2} \approx 363\ \mu\text{V}_{\text{rms}} \approx \sqrt{\left(128 * 2.8\ \mu\text{V}_{\text{rms}}\right)^2}$$

In this case the total output noise is well-approximated (< 1.5% difference) by simply multiplying the RTI noise over the specified bandwidth by the pre-filter gain.

More generally, the total output noise over the same 100 kHz bandwidth with PRG gain factor G_{PR} and a POG gain factor G_{PO} is estimated as

$$E_n = \sqrt{\left(G_{PR} G_{PO} E_{RTI}\right)^2 + \left(E_{RTO}\right)^2} \tag{13}$$

If the low-pass filter is included in the signal path and set to a cutoff frequency f_c, then (13) must be recast as

$$E_n = \sqrt{\left(G_{PR} G_{PO} e_{RTI} \sqrt{k f_c}\right)^2 + \left(E_{RTO}\right)^2} \tag{14}$$

where k is the brickwall correction factor for the filter.

The reader should bear in mind a number of caveats related to (13) and (14). A portion of the noise embedded in the E_{RTO} term of (13) and (14) is passed through the filter and amplified by the POG, but the effect is omitted here for the sake of simplifying the calculations. In addition, the 100 kHz reference bandwidth does not exactly equate to the full bandwidth of the signal conditioner (e.g., for a standard 28144 conditioner, the bandwidth is 500 kHz). And a more complete model would account for the effect of the low-pass filter setting on the noise at the output.

5 As calculated using (11): $\approx 9\,\text{nV}/\sqrt{\text{Hz}} * \sqrt{100\ \text{kHz}} \approx 2.8\ \mu\text{V}_{\text{rms}}$

3.4. Estimating SNR

In measurement applications, the level of self-noise in a signal conditioner should be evaluated relative to the expected full-scale signal level. This is generally expressed as a signal-to-noise (SNR) ratio given in decibels (dB):

$$SNR = 20\log_{10}\left(\frac{y_{out}}{E_n}\right) \tag{15}$$

where y_{out} is the full-scale RMS voltage of the signal at the output. It's important to remember that (15) defines the SNR for self-noise only: a total SNR for the measurement system would include external noise and sensor noise in RSS combination with E_n.

Consider the following example. The expected in-band signal input to the conditioner from an attached sensor is 100 mV$_p$. To generate an output signal at the maximum input level of an attached ADC (= 10 V$_p$), a total gain of 100 is applied as PRG (i.e. G_{PR} = 100, G_{PO} = 1). The signal conditioner noise is equivalent to the specifications listed above for PFI's 28144 system. PFI's LP4P filter with a cutoff frequency f_c of 25 kHz and a brickwall correction factor k of 1.11 (see Table 1) is included in the signal path. With this setup, the total self-noise on the signal conditioner output is estimated using (14):

$$E_n \approx \sqrt{\left((100)(9\text{x}10^{-9}\text{V}_{\text{rms}}/\sqrt{\text{Hz}})\sqrt{(1.11)(25\text{x}10^{3}\text{Hz})}\right)^2 + (60\text{x}10^{-6}\text{V}_{\text{rms}})^2}$$

$$E_n \approx 161\ \mu\text{V}_{\text{rms}}$$

The self-noise SNR of the measurement can then be calculated with (15):

$$SNR \approx 20\log_{10}\left(\frac{\frac{(100)(100\text{x}10^{-3}\text{V}_{\text{p}})}{\sqrt{2}}}{161\text{x}10^{-6}\text{V}_{\text{rms}}}\right)$$

$$SNR \approx 93\ \text{dB}$$

This means that the input signal from the sensor is approximately 93 dB above the noise floor of the signal conditioner.

4. Summary

Analog signal conditioners are a critical component of high-performance measurement systems. The circuit components that provide amplification and filtering ensure the analog signals of interest are clean and ready for digital conversion. Yet they also introduce electrical noise into the signal chain. Though generally small in magnitude, this noise should be quantified by system manufacturers and understood by measurements engineers. PFI's noise specifications are based on a simplified model that, when combined with a basic understanding of noise characterization, can be used to quickly and easily estimate the signal-to-noise ratio for a given measurement.

A. Appendix

A1. Statistical Description of Noise

Consider a noise voltage waveform such as that shown in Figure 1. The random voltage at time t is $v(t)$. Over a measurement interval of duration T, the mean voltage is defined by

$$\overline{v(t)} = \frac{1}{T}\int_0^T v(t)dt \tag{A.1}$$

The deviation of the noise voltage about the mean is

$$v'(t) = v(t) - \overline{v(t)} \tag{A.2}$$

The standard deviation of the noise voltage waveform is expressed in terms of $v'(t)$:

$$\sigma = \sqrt{\frac{1}{T}\int_0^T \left(v'(t)\right)^2 dt} \tag{A.3}$$

Hence for a zero-mean noise waveform, the standard deviation is just:

$$\sigma = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt} \tag{A.4}$$

which, referring to (A.1), is just the RMS voltage:

$$\sigma = \sqrt{\overline{v(t)^2}} \tag{A.5}$$

A2. Combining Independent Noise Sources

Consider two random, independent (uncorrelated) zero-mean noise voltages $v_1(t)$ and $v_2(t)$. The RMS voltage of their sum is expressed as

$$v_{rms} = \sqrt{\overline{\left(v_1(t) + v_2(t)\right)^2}} \tag{A.6}$$

Expanding (A.6) gives

$$v_{rms} = \sqrt{\overline{v_1(t)^2} + \overline{v_2(t)^2} + \overline{2v_1(t)v_2(t)}} \tag{A.7}$$

For uncorrelated noise,

$$\overline{2v_1(t)v_2(t)} = 0 \tag{A.8}$$

leaving

$$v_{rms} = \sqrt{\overline{v_1(t)^2} + \overline{v_2(t)^2}} = \sqrt{v_{1,rms}^2 + v_{2,rms}^2} \qquad \text{(A.9)}$$

That is, the RMS voltage of each independent noise source adds as the root-sum-square (RSS) to produce a combined RMS noise voltage v_{rms}. More generally,

$$v_{rms} = \sqrt{\sum_{i=1}^{N} v_{i,rms}^2} \qquad \text{(A.10)}$$

for any N independent sources.

A3. Pink and White Noise Combined Spectral Density

Consider two noise sources, one pink and one white, with noise spectral densities e_{nf} and e_{nw}, respectively, as defined in (4) and (5). Over a bandwidth B defined by a lower frequency bound f_L and an upper frequency bound f_H, these sources add in RSS fashion (as described in Section A.2) to yield a combined RMS voltage:

$$E_n = \sqrt{\int_{f_L}^{f_H} \frac{K^2}{f} df + \int_{f_L}^{f_H} e_c^2 df} \qquad \text{(A.11)}$$

Making use of the corner frequency f_{cr} defined by (6), (A.11) can be rewritten as

$$E_n = \sqrt{\int_{f_L}^{f_H} e_c^2 \left(\frac{f_{cr}}{f} + 1 \right) df} = \sqrt{\int_{f_L}^{f_H} e_n^2 df} \qquad \text{(A.12)}$$

where the combined spectral density is defined as in (9):

$$e_n = e_c \sqrt{\frac{f_{cr}}{f} + 1} \qquad \text{(A.13)}$$

This is the same relation as (7) in Section 2.3.

A4. Noise Analysis Over a Finite Bandwidth

The noise spectral density e_n defined by (A.13) has no upper frequency limit: it implies a montonically increasing total RMS noise with increasing bandwidth. To account for rolloff of the noise amplitude due to the bandwidth-limiting effect of the filters and amplifiers in a measurement circuit, (A.13) must be modified by an appropriate frequency response function.

Consider a noise spectral density that rolls off with a Butterworth-type response of order n. For this case, (A.13) becomes

$$e_n(f) = \frac{e_c \sqrt{\left(\frac{f_{cr}}{f} + 1 \right)}}{\sqrt{1 + \left(\frac{f}{f_c} \right)^{2n}}} \qquad \text{(A.14)}$$

where f_c is the -3 dB cutoff frequency[6]. The total RMS noise can be determined via integration of (A.14) from a finite low frequency f_L to ∞ using (9):

$$E_n = \sqrt{\int_{f_L}^{\infty} e_n^2 df} \qquad \text{(A.15)}$$

Substituting (A.14) into (A.15) and assuming $f_c >> f_L$, the integral can be expressed as

$$E_n^2 = \int_{f_L}^{\infty} \frac{e_c^2 f_{cr}}{f\left(1+\left(\frac{f}{f_c}\right)^{2n}\right)} df + \int_{0}^{\infty} \frac{e_c^2}{1+\left(\frac{f}{f_c}\right)^{2n}} df - \int_{0}^{f_L} e_c^2 df \qquad \text{(A.16)}$$

which yields

$$E_n = e_c \sqrt{\frac{f_{cr}}{2n} \ln\left(\frac{f_c^{2n}}{f_L^{2n}}+1\right) + kf_c - f_L} \qquad \text{(A.17)}$$

where f_{cr} is the corner frequency defined by (6) and k is the brickwall correction factor that depends on n. Analytical expressions and numerical values for k are given in Table A1 for the Butterworth response represented by (A.17).

Relations (10) and (12) in Sections 2.3 and 2.4 can be obtained from (A.17) by considering the appropriate limits. Ignoring the high-frequency rolloff is equivalent to letting $n \to \infty$ in (A.17):

$$\lim_{n\to\infty}\left(e_c \sqrt{\frac{f_{cr}}{2n} \ln\left(\frac{f_c^{2n}}{f_L^{2n}}+1\right) + kf_c - f_L} \right) = e_c \sqrt{f_{cr} \ln\left(\frac{f_c}{f_L}\right) + \left(f_c - f_L\right)} \qquad \text{(A.18)}$$

Note that the limit in (A.18) implies that $k \to 1$ and thus f_c is a brickwall frequency equivalent to f_H in (10), giving a bandwidth $B = f_c - f_L = f_H - f_L$.

Considering (A.17) in the broadband case, where it's assumed $f_c >> f_{cr}$ and $f_c >> f_L$, leads to the approximation given in (12):

$$E_n \approx e_c \sqrt{kf_c} \qquad \text{(A.19)}$$

Butterworth Filter Order (n)	Table A1. Brickwall Correction Factors (k)	
	Definition	**Value**
1	$\pi/2$	1.571
2	$\pi/\left(2\sqrt{2}\right)$	1.111
3	$\pi/3$	1.047
4	$\pi/\left(4\sqrt{2-\sqrt{2}}\right)$	1.026
6	$\pi/\left(3\sqrt{2}\left(\sqrt{3}-1\right)\right)$	1.012
8	$\pi/\left(8\sqrt{2-\sqrt{2+\sqrt{2}}}\right)$	1.007

6 The plot of e_n in Figure 6 corresponds to (A.14) with $f_{cr} = 0$, $n = 1$, and $f_c = 500$ kHz.

PRECISION FILTERS, INC.

240 Cherry Street
Ithaca, New York 14850

607-277-3550
www.pfinc.com